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Mass generation and the dynamical role of the Katoptron Group

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Abstract

Heavy mirror fermions along with a new strong gauge interaction capable of breaking the electroweak gauge symmetry dynamically were recently introduced under the name of katoptrons. Their main function is to provide a viable alternative to the Standard-Model Higgs sector. In such a framework, ordinary fermions acquire masses after the breaking of the strong katoptron group which allows mixing with their katoptron partners. The purpose of this paper is to study the elementary-scalars-free mechanism responsible for this breaking and its implications for the fermion mass hierarchies.

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1 Motivation

Identifying the nature of the various symmetries which might be at the source of the observed fermion-mass pattern constitutes one of the central investigation goals in high-energy physics [1]. The breaking of an horizontal symmetry for instance was used some time ago in an attempt to explain the mass hierarchy between fermion generations [1]. The katoptron theory that was recently introduced in [2] contains a strongly interacting "horizontal" gauge symmetry acting on a new fermion sector. It constitutes a dynamical alternative to the Standard-Model (SM) Higgs mechanism and in parallel addresses the strong CP problem. Strong dynamics render the study of such theories quite difficult, but in parallel illuminate the inner works of mass generation.

On the other hand, deciding to tackle instead exclusively perturbative problems yielding precise mathematical results would be misleading, since exact solvability is not a physics goal *per se*, whereas the correct understanding of physical phenomena is. As reminds us Herakleitos from 6th century B.C., "nature likes to hide", and physics gives frequently rise to highly non-trivial phenomena defying rigorous analytical description, quark confinement being a well-known example.

The motivation for the analysis of the katoptron model is basically two-fold. One comes from experimental data [3] suggesting the existence of new particles close to the weak scale and thus accessible to the next generation of experiments in the not-too-distant future. The other one stems from considerations related to a particular unification of all particle interactions at energy scales which are high

[1] "Of symmetries indeed, we consider the small which we perceive, we neglect however the principal and greatest" wrote Plato in his dialog "Timaios".

enough to naturally suppress proton decay. This was discussed in [4] and placed in a higher-dimensional unified framework in [5].

To prove analytically and demonstrate explicitly that the katoptron model does indeed solve the hierarchy problem *via* this unification in its essence, contrary to other currently popular approaches, one should write down precise mathematical formulas, which were already implied in [4] for the symmetry breaking channel $SU(4)_{PS} \times SU(2)_R \longrightarrow SU(3)_C \times U(1)_Y$, relating the various energy scales of the model with each other. The starting point lies in the renormalization-group equations that give rise to the running of a given gauge coupling g with momentum p, which is described, to 1-loop, by the well-known equation

$$\alpha^{-1}(p) = \alpha^{-1}(p_0) + c \ln{(p/p_0)}, \tag{1}$$

where $\alpha = \frac{g^2}{4\pi}$, p_0 is some reference scale and $c = \frac{11N - 2N_f}{6\pi}$ when the theory contains N_f fermions transforming under the fundamental representation of the gauge group $SU(N)$.

In order to derive analytical formulas for the physical scales of the model, one has to use the following relations having their source in unification constraints and dynamical assumptions [4] defining in parallel the scales Λ_{PS}, Λ_{QCD}, Λ_K (denoted by Λ_M in [4]) and Λ_{GUT} respectively:

$$\begin{aligned} \alpha_Y^{-1}(\Lambda_{PS}) &= \frac{3\alpha_L^{-1}(\Lambda_{PS}) + 2\alpha_C^{-1}(\Lambda_{PS})}{5} \\ \alpha_C^{-1}(\Lambda_{QCD}) \sim 1, &\qquad \alpha_K^{-1}(\Lambda_K) \sim 1 \\ \alpha_L(\Lambda_{GUT}) = \alpha_{PS}(\Lambda_{GUT}) &= \alpha_K(\Lambda_{GUT}) \equiv \alpha_{GUT}, \end{aligned} \tag{2}$$

where $\alpha_{Y,L,C,PS,K}$ are the couplings corresponding to the gauge groups $U(1)_Y$,

$SU(2)_L$, $SU(3)_C$, $SU(4)_{PS}$ and $SU(3)'$, the katoptron generation gauge group (denoted by $SU(3)_{2G}$ in [4]).

The definition of the following constants proves then to be useful:

$$A = \alpha_Y^{-1}(M_Z) \quad - \quad \frac{1}{5}\left(3\alpha_L^{-1}(M_Z) + 2\alpha_C^{-1}(M_Z)\right) \tag{3}$$

$$c_N = \frac{11N-12}{6\pi}, \qquad \tilde{c_N} = \frac{11N-24}{6\pi} \text{ for } N = 0,2,3,4 \tag{4}$$

$$c_K = \frac{17}{6\pi}, \qquad B = \frac{3c_2+2c_3}{5} - c_0 \tag{5}$$

$$E = \alpha_C^{-1}(M_Z) - 1 + (1-\alpha_L^{-1}(M_Z))\frac{\tilde{c_4}-c_K}{\tilde{c_2}-c_K} + \frac{A(\tilde{c_3}-\tilde{c_4})}{B} \tag{6}$$

$$F = \frac{c_2(\tilde{c_4}-c_K)}{\tilde{c_2}-c_K} + \tilde{c_3} - \tilde{c_4} - c_3, \tag{7}$$

where A is expressed in terms of the values of the gauge couplings measured at the mass M_Z of the Z^0 gauge boson, c_K describes the running of α_K, and the values $N = 0,2,3,4$ for $c_N, \tilde{c_N}$ correspond to the couplings $\alpha_{Y,L,C,PS}$ for scales below and above Λ_K respectively [2].

All the necessary ingredients are now available in order to express the physical scales of the model in a 1-loop approximation. Apart from the easily-derived $\Lambda_{QCD} = M_Z \exp\left(\frac{1-\alpha_C^{-1}(M_Z)}{c_3}\right)$, assuming that α_K does not influence consider-

[2] In order to derive order-of-magnitude relations, it is assumed here that all katoptrons decouple at scales below Λ_K. It will be shown later that a relatively small mass hierarchy between katoptrons renders this not exactly true.

ably the rest of the gauge couplings when it becomes strong [4] and noting that $\tilde{c_2} - c_K \sim -1$, one has

$$\Lambda_{GUT} = \Lambda_K \left(\exp\left(1 - \alpha_L^{-1}(M_Z)\right) \left(\frac{M_Z}{\Lambda_K} \right)^{c_2} \right)^{\frac{1}{\tilde{c_2} - c_K}} \sim \Lambda_K e^{\alpha_L^{-1}(M_Z)} \tag{8}$$

$$\Lambda_{PS} = M_Z e^{A/B} \tag{9}$$

$$\Lambda_K = M_Z e^{E/F}. \tag{10}$$

In these relations, input variables are only the three coupling constants $\alpha_{Y,L,C}(M_Z)$ and M_Z. Using the experimentally measured values $\alpha_Y^{-1}(M_Z) \sim 59.2$, $\alpha_L^{-1}(M_Z) \sim 29.6$, $\alpha_C^{-1}(M_Z) \sim 8.4$ and $M_Z \sim 91.2$ GeV [6], one obtains $\Lambda_{QCD} \sim 0.12$ GeV, $\Lambda_{GUT} \sim 5.6 \times 10^{15}$ GeV, $\Lambda_{PS} \sim 6 \times 10^{13}$ GeV, and $\Lambda_K \sim 840$ GeV, which are of course consistent with the results of [4]. If one claims further that the value of M_Z is determined dynamically by Λ_K, one is left with only three independent input constants.

To understand how one can argue this, remember that M_Z can be expressed by

$$M_Z = v\sqrt{\pi(\alpha_Y(M_Z) + \alpha_L(M_Z))}, \tag{11}$$

where $v \sim 250$ GeV denotes the weak scale. If this scale is generated dynamically, it is approximately given by [2]

$$v = \frac{1}{2\pi}\sqrt{\sum_i M_i^2 \ln\left(\Lambda_\chi / M_i\right)}, \tag{12}$$

where i counts the new fermion electroweak doublets introduced in the theory and $\Lambda_\chi \sim \Lambda_K$ is the katoptron chiral symmetry breaking scale. In the following section one can see that the contribution of the eight lighter of the twelve new doublets of the katoptron model in this relation is negligible, and that to avoid fine-tuning one may take the scales Λ_χ and M_i to be of the same order of magnitude. In this case one has $v \lesssim \Lambda_K/\pi$, a result consistent with the Z-boson mass, a mass which is therefore no longer independent from Λ_K.

Alternatively, one could suppose that there exists a more fundamental theory producing values for α_{GUT}, Λ_{GUT} and Λ_{PS}, in which case the values of the gauge couplings at M_Z and the rest of the scales would be the output of the model. Anyway, the fact that this theory allows for the determination of unique order-of-magnitude energy-scale relations such as the ones just given and for the transparent solution of the hierarchy problem constitutes a quite powerful motivation for the further study of katoptron dynamics in order to resolve correctly the puzzle of fermion mass generation and open the "black box" of fermion Yukawa couplings.

2 Gauge-symmetry breakings and the fermion masses

2.1 Self-breaking of $SU(3)'$

We start by listing the low-energy particle content of the theory [2] since it proves useful for the discussion of this section. Under the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)'$, fermions transform as

SM fermions	Katoptrons
$q_L : (\mathbf{3},\ \mathbf{2},\ 1/3,\ \mathbf{1})_i$	$q_R^K : (\mathbf{3},\ \mathbf{2},\ 1/3,\ \mathbf{3})$

$$
\begin{array}{ll}
l_L : (\mathbf{1},\ \mathbf{2},\ -1,\ \mathbf{1})_i & l_R^K : (\mathbf{1},\ \mathbf{2},\ -1,\ \mathbf{3}) \\
q_R^c : (\bar{\mathbf{3}},\ \mathbf{1},\ {}^{-4/3}_{+2/3},\ \mathbf{1})_i & q_L^{K\,c} : (\bar{\mathbf{3}},\ \mathbf{1},\ {}^{-4/3}_{+2/3},\ \mathbf{3}) \\
l_R^c : (\mathbf{1},\ \mathbf{1},\ {}^{0}_{2},\ \mathbf{1})_i & l_L^{K\,c} : (\mathbf{1},\ \mathbf{1},\ {}^{0}_{2},\ \mathbf{3}),
\end{array}
\tag{13}
$$

where q and l denote quarks and leptons respectively and $i = 1, 2, 3$ is a SM-generation index. It should be reminded here that in the unified context of [5] this theory is anomaly-free. The katoptron coupling α_K becomes strong at energy scales around Λ_K and the katoptrons acquire dynamical masses in a similar way that ordinary quarks acquire dynamical ("constituent") masses because of QCD.

Therefore, the condensate [3] $< \bar{\psi}_L^K \psi_R^K >$ breaks not only the electroweak gauge symmetry but also the katoptron chiral symmetry at the right scale, substituting thus the elementary Higgs mechanism. In addition, the same condensate is presumed to cause the self-breaking of the katoptron generation group $SU(3)'$ *via* the channel $\mathbf{3} \times \mathbf{3} \longrightarrow \bar{\mathbf{3}}$, where $\mathbf{3}$ denotes the fundamental representation of $SU(3)'$ in which katoptrons reside. As is shown in the next subsection, the formation of this condensate proves to be very crucial also for the generation of SM-fermion masses.

The channel $\mathbf{3} \times \bar{\mathbf{3}} \longrightarrow \mathbf{1}$ would leave the katoptron generation symmetry intact, but it cannot be realized with Lorenz-scalar operators (contrary to what happens in QCD, here both ψ_R^K and $\psi_L^{K\,c}$ reside in the $\mathbf{3}$ of $SU(3)'$). Therefore, assuming that gauge symmetries cannot break Lorenz symmetries, the katoptron group most likely self-breaks. Majorana masses are also not expected to be gener-

[3] The complex conjugate parts of composite fermionic operators are omitted here and in the following for simplicity. The influence of weak interactions on the dynamics discussed is therefore neglected in this first approach. Note also that specifying the correct fermion handedness is indispensable for the gauge invariance of these operators.

ated, since QCD interactions make the $SU(3) \times U(1)_Y$ symmetry-preserving condensate which produces Dirac masses correspond to the most attractive channel.

The self-breaking of $SU(3)'$ indicates that third-generation katoptrons (denoted by a "3" superscript below) acquire masses on the order of Λ_K *via* the condensate $< \bar{\psi_L}^{3K} \psi_R^{3K} >$, justifying thus the approximate relation giving the weak scale in the previous section. The generation group is consequently broken down to $SU(2)'$, and the corresponding coupling also becomes strong in its turn but at lower energies. To calculate the scale at which the new chiral symmetry breaking related to the two lighter katoptron generations takes place, recall that this happens when the relevant gauge coupling reaches the critical value $\alpha_c = \frac{\pi}{3C_2(R)}$ [7], where $C_2(R)$ is the quadratic Casimir of the representation R of the gauge group. For fermions transforming under the fundamental representation of an $SU(N)$ gauge group, C_2 is given by $C_2 = \frac{N^2-1}{2N}$.

Denoting the critical couplings and chiral symmetry breaking scales of $SU(3)'$ and $SU(2)'$ by α_c and $\tilde{\alpha_c}$ and by Λ_χ and $\tilde{\Lambda_\chi}$ respectively, one has

$$\tilde{\alpha_c^{-1}} = \alpha_c^{-1} + \tilde{c_K} \ln{(\tilde{\Lambda_\chi}/\Lambda_\chi)} \tag{14}$$

where $\tilde{c_K} = 1/\pi$ describes the running of the $SU(2)'$ coupling. This relation yields $\Lambda_\chi = \tilde{\Lambda_\chi} e^{7/4} \sim 5.75\tilde{\Lambda_\chi}$, which should also express approximately the mass hierarchy between the third and the two lighter katoptron generations.

In view of the fact that the values of the critical couplings are quite large, the 1-loop β-function is not very accurate and the equation above should only be considered as a crude approximation giving order-of-magnitude results. Furthermore, note that the condensates $< \bar{\psi_L}^{(1,2)K} \psi_R^{(1,2)K} >$ which break the chiral symmetry of

the two lighter katoptron generations do not break the $SU(2)'$ group. On the other hand, QCD interactions can break the remaining katoptron generation symmetry at lower energies by forming condensates of the form $<\bar{q}_L^K q_R>$ and having Λ_{QCD}^3 as a natural order of magnitude.

2.2 Mass hierarchies and mixing angles

Having described the main qualitative features of the dynamics of the katoptron group, we are ready to start an order-of-magnitude calculation of the SM-fermion masses and mixing angles. To understand why the third-generation standard-model fermions are much heavier than the other ones, one has to study the relevant operators in the effective Lagrangian. What is clearly needed is the formation of multi-fermion composite operators containing fermion bilinears $\bar{\psi}_L^K \psi_R$ which mix katoptrons with SM fermions and thus provide a mass feed-down (generalized see-saw) mechanism.

Since the original Lagrangian is chirally symmetric, the search is focused on non-renormalizable operators arising non-perturbatively. These might not be generated explicitly by gauge interactions, but they should be consistent with the gauge symmetries of the model. Unlike extended-technicolour operators in technicolour theories, these operators are not likely to be highly suppressed since the energy scale where the katoptron symmetry is broken is obviously very close to the scale where the katoptron coupling becomes strong.

Simple inspection of familiar types of operators is initially discouraging. The operator $\frac{1}{\Lambda_{QCD}^2}(\bar{\psi}_R \psi_L^K)\bar{\psi}_L^K \psi_R$ for instance is not supported by dynamics strong enough to generate third-generation SM fermion masses, since $\frac{1}{\Lambda_{QCD}^2} < \bar{\psi}_R \psi_L^K >$

should be on the order of $\Lambda_{QCD} \ll m_{t,b}$ and is more suited for lighter SM-fermion masses. Moreover, an operator of the form $\frac{1}{\Lambda_K^2}(\bar{\psi}_L^K \psi_R^K)\bar{\psi}_L^K \psi_R$ would have strong enough dynamics, since $\frac{1}{\Lambda_K^2} < \bar{\psi_L}^K \psi_R^K >\sim \Lambda_K$, but unfortunately is not gauge invariant and would break the electroweak symmetry explicitly.

One is therefore lead to study higher-dimensional composite operators. The list of the ones quoted next is meant to be indicative and by no means exhaustive. Consider for instance an operator of the form $\frac{1}{\Lambda_K^5}(\psi_R^{\bar{3}K}\psi_L^{3K})(\psi_R^{\bar{3}K}\psi_L^{3K})\psi_L^{\bar{3}K}\psi_R^a$, where a stands for the various SM fermions. When the katoptron gauge coupling α_K becomes strong, katoptron condensates are formed and one has dynamical mass terms in the effective Lagrangian of the form $m_{3a}\bar{\psi}_L^{3K}\psi_R^a$, where

$$m_{3a} = \frac{\lambda_{3a}}{\Lambda_K^5} < \psi_R^{\bar{3}K}\psi_L^{3K} >^2 \tag{15}$$

and λ_{3a} is an effective multi-fermion coupling. Since $< \psi_R^{\bar{3}K}\psi_L^{3K} >\sim \Lambda_K^3$, one finds that $m_{3a} \sim \lambda_{3a}\Lambda_K$. Note that the operator above is gauge-invariant only when it involves katoptrons of the third generation. This makes mass terms of the form $\bar{\psi}_L^K \psi_R$ much larger for third-generation katoptrons, while for the lighter generations these are *a priori* expected to be quite smaller.

This discussion renders the connection between the heaviness of the top quark and the possibly large δg_R^t [2] clearer. When katoptron condensates are formed, the non-perturbative operator $\frac{1}{\Lambda_K^8}(\psi_L^{\bar{3}K}\psi_R^{3K})(\psi_L^{\bar{3}K}\psi_R^{3K})\psi_L^{\bar{3}K}\psi_R^a\psi_L^{\bar{3}K}\psi_R^a$ (the lowest-dimensional gauge-invariant operator relevant to δg_R^t which involves katoptron dynamics) becomes proportional to $\frac{m_{3a}}{\Lambda_K^3}\psi_L^{\bar{3}K}\psi_R^a\psi_L^{\bar{3}K}\psi_R^a$. However, this operator is conjectured to be responsible for the deviation of the weak couplings $g_R^{t,b}$ from their standard-model values and consequently for the smallness of the S parameter

[2]. Since the mass relation $m_{t^K} \sim m_{b^K}$ is expected to hold, the fact that $m_t \gg m_b$ translates into $m_{3t} \gg m_{3b}$ within the generalized see-saw framework of the model, and explains why the anomalous-coupling relation $\delta g_R^t \gg \delta g_R^b$ is plausible.

A similar higher-dimensional operator could make electroweak-invariant mass terms not involving third-generation katoptrons larger than the naively supposed Λ_{QCD} scale. Consider for instance the operator $\frac{1}{\Lambda^8}(\psi_L^{\bar{3}K}\psi_R^{3K})(\psi_R^{\bar{3}K}\psi_L^{3K})(\psi_L^{\bar{2}K}\psi_R^a)\psi_L^{\bar{2}K}\psi_R^a$, where Λ is some relevant energy scale. At low-enough energies, we have the formation of condensates due not only to QCD but to katoptron interactions as well. The operator then becomes $\frac{1}{\Lambda^8} < \psi_L^{\bar{3}K}\psi_R^{3K} >< \psi_R^{\bar{3}K}\psi_L^{3K} >< \psi_L^{\bar{2}K}\psi_R^a > \psi_L^{\bar{2}K}\psi_R^a \sim m_{2a}\psi_L^{\bar{2}K}\psi_R^a$ with $m_{2a} = \lambda_a \Lambda_{QCD}^{1-\epsilon_a}\Lambda_K^{\epsilon_a}$, where the parameters λ_a and ϵ_a should be determined by the non-perturbative dynamics of the model.

It is apparent that these mass terms could be as large as Λ_K according to the values assumed by ϵ_a, the computation of which lies however beyond the scope of this letter. It might just be added that in principle ϵ_a is momentum-dependent, and a simple relevant ansatz would be $\epsilon_a = \gamma_a \ln(p_0/p)$, with γ_a an appropriate positive anomalous dimension. The appearance of such terms is not surprising, since no symmetry protects these masses from being large after the katoptron generation group is broken. The fact that the $SU(2)\prime$ coupling is already strong just before this symmetry is broken could accentuate this effect. Unfortunately, nature has not given us yet examples of gauge symmetries broken below the scale where their couplings become strong and the relevant dynamics are hard to pin down. It is assumed next that ϵ_a is particularly large for third-generation SM fermions, something that proves to be convenient for the numerical exercise below.

The mass-matrix example given in [2] can now be improved by considering

merely for illustration purposes the following mass matrices, which for simplicity are taken to be real and have the form

$$\mathcal{M}_i = \begin{pmatrix} 0 & m_i \\ m_i & M_i \end{pmatrix}, i = U, D \tag{16}$$

for the up-type $(U)$and down-type quarks (D), with M_i, m_i symmetric (similar matrices can be constructed for leptons [4]):

$$m_U(\mathrm{GeV}) = \begin{pmatrix} 0.7 & 0.8 & 15 \\ 0.8 & 0.8 & 73 \\ 15 & 73 & 410 \end{pmatrix} \quad , \quad m_D(\mathrm{GeV}) = \begin{pmatrix} 0.7 & 0.8 & 1 \\ 0.8 & 0.8 & 15 \\ 1 & 15 & 45 \end{pmatrix}. \tag{17}$$

The dynamical assumption is made again that the $SU(2)_L \times U(1)_Y$-breaking katoptron dynamical mass submatrices are diagonal and have the form

$$M_U = M_D(\mathrm{GeV}) = \begin{pmatrix} 170 & 0 & 0 \\ 0 & 170 & 0 \\ 0 & 0 & 1000 \end{pmatrix}. \tag{18}$$

Note that the matrices $M_{U,D}$ are taken to be equal to each other in order to respect isospin symmetry and avoid large contributions to the T parameter, since the $m_t - m_b$ hierarchy is reproduced by terms in the $m_{U,D}$ matrices which are electroweak singlets. The hierarchy between the third and the two lighter katoptron generations is consistent with our previous discussion and interestingly enough proves to be crucial for the correct reproduction of the SM-CKM matrix. Moreover, the terms in $m_{U,D}$ not involving third-generation fermions do not exceed the GeV scale, in accordance with the previous considerations.

The rigorous study of the dynamics producing these specific mass entries corresponding to particular values of ϵ_a and λ_a introduced before, and which give rise to the large mass hierarchy between up- and down-type quarks or for instance

between the quarks and leptons in the SM requires a lengthier investigation. It is interesting to note however that near-critical interactions in dynamical-symmetry-breaking theories may typically reproduce such hierarchies [8]. The considerations in that reference are readily applicable here as well, since the electroweak-invariant mass terms that mix SM-fermions with their partners and give them mass are generated by critical dynamics which are in addition responsible for the katoptron-symmetry-breaking condensate.

After diagonalisation of the mass matrices, the quark masses are found to be approximately equal (in GeV units and renormalized at TeV scales) to

$$165, 0.75, 0.001 \qquad \mathrm{Up-type\ SM\ quarks} \tag{19}$$

$$170, 184, 1152 \qquad \mathrm{Up-type\ katoptron\ quarks} \tag{20}$$

$$3.5, 0.06, 0.003 \qquad \mathrm{Down-type\ SM\ quarks} \tag{21}$$

$$170, 171, 1002 \qquad \mathrm{Down-type\ katoptron\ quarks} \tag{22}$$

which, along with the corresponding leptons, reproduce a correct order of magnitude for the weak scale. Therefore, as regards particles carrying QCD color, this model predicts the existence of four new fermions not much heavier than the top quark, and two more with masses around 1 TeV.

The mass matrices introduced give rise to a unitary generalized CKM matrix describing the mixing between the fermions of the theory, with a non-unitary sub-matrix V_{SM-CKM} corresponding to the standard-model fermions given (in absolute values) by

$$|V_{SM-CKM}| \sim \begin{pmatrix} 0.97 & 0.23 & 0.008 \\ 0.22 & 0.97 & 0.07 \\ 0.005 & 0.06 & 0.95 \end{pmatrix}, \tag{23}$$

which is reasonably close to the experimentally measured SM-CKM matrix (obviously not assuming SM-CKM unitarity and taking renormalization to higher scales into account). Note the small predicted value of $|V_{tb}| \sim 0.95$ which is due to the large mixing of the top quark with its katoptron partner and is obviously related to the large value of m_t.

3 Conclusions

Previous attempts to introduce katoptrons in physics beyond the SM had already made clear how electroweak symmetry can be broken dynamically by non-perturbative effects. However, the source of SM-fermion masses was still left rather obscure. The present work constitutes a first attempt to clarify certain qualitative aspects of fermion mass generation in connection with the katoptron gauge-group self-breaking and to identify the reasons for the appearance of various mass hierarchies with no recourse to unnaturally light elementary scalar particles. A detailed computation of fermion masses within this framework would entail a big effort tackling strong-dynamics issues, but in parallel would complete the picture of katoptron theory.

Moreover, apart from its theoretical consistency the model possesses clear experimental signatures which place it on solid epistemological grounds and deserve to be looked at in future experimental projects. [4]. If the scheme proposed finally proves to be true, it would be the first known case in nature of a gauge symmetry breaking itself and another gauge symmetry due to its non-perturbative dynamics. Mass as a physical quantity would therefore have its source in gauge interactions;

[4] A section of a letter written by the philosopher Epicure to Pythocles more than two millennia ago is enlightening at this point: "But when one accepts one theory and rejects another which harmonizes just as well with the phenomenon, it is obvious that one altogether leaves the path of scientific enquiry and has recourse to myth"

and the mass of the presently known elementary particles would be a manifestation of mixing with their - now merely virtual - katoptron partners, since these must have decayed just after the creation of our world.

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